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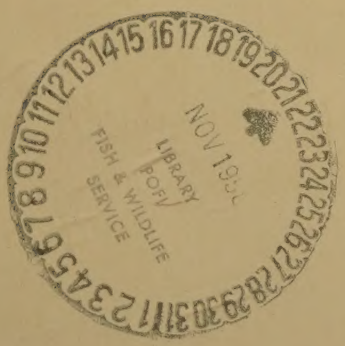

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変態を完了した半数性トノサマガエルについて

宮　田　澄　男

On a Completely Metamorphosed Haploid Frog, *Rana nigromaculata* HALLOWELL

Sumio MIYADA

無尾類の半数体は生活力が極めて弱く，採餌して成長を続ける個体を得ることは非常に困難とされていた．そのため，現在迄に変態を完了した半数性幼蛙に関する報告はまだ見られない．

筆者は，さきに[1] トルイヂン・ブルー処理精虫での媒精卵から発生するトノサマガエル半数体幼生の中に，採餌する個体が存在するのを見出し，幼蛙を得る可能性を指摘したが，1955 年初めて変態期に達した 1 匹の半数体を得ることができた[2]．もし多数の半数性幼蛙を得ることができれば，遺伝学的にも細胞生理学的にも興味ある問題が見出されると考え，1956 年，半数性幼蛙の大量獲得を目的とする実験を開始した．その結果，同年の実験で，1 匹の変態完了個体を得る迄に成功したので，その概略を報告する．本研究について，終始御懇切な御指導と御鞭達並びに本稿の御校閲を頂いた，広島大学川村智治郎教授に対し深謝の意を表する．

材料及び方法

実験に使用した卵は，4 月下旬から 5 月上旬までの間は，野外で抱接中のトノサマガエルから得，5 月下旬に使用した卵は，未排卵ダルマガエルの雌に脳下垂体を移植して得た．雄はすべてトノサマガエルを使用した．半数体作製の具体的な方法は，さきに報告した方法[1] に準じて行つたが，本実験では，トルインヂン・ブルー濃度は，有効範囲のものを適宜使用し，濃度と発生率の関係等については特別な考慮を払わなかつた．実験区の卵は，同じい親の卵で正常受精を行つた対照区のものと同一器内で発生させ，両者の発生経過を比較観察した．孵化期前後に到り，実験区の幼生で水腫を生じて来たもの及び畸型幼生は，実験の目的上保存せず，逐次放棄した．孵化後約 2 週間で，採餌しはじめたオタマジャクシは，逐次尾端法によつて染色体数を算定し，半数性個体を同定した．良好な中期分裂像を発見できなかつた個体は，尾の再生を待つて再検すべきであつたが，非常に生活力が弱く，尾の反復切断は死を招くおそれがさつたので，倍数性の一般的同定法[3][4] によつて，休止核の大きさと分布密度及び赤血球の大きさを二倍体と比較して決定した．

実験結果

(1) 発生経過

a) 孵化期迄．　作製実験は前後 8 回，計 14 匹の雌を使用した．このうち，7 例 13 匹の雌の卵からは，生存半数体＊は全く得られず，すべて典型的半数体＊もしくわ畸型で，早期に死滅した．結局 5 月 24 日に行つた最後の作製実験によつて，媒精卵 385 個から 96 匹の生存半数体を得た．これ等半数体は，尾芽期迄は発生速度等に関しては，他の実験区と殆ど差異が認められず，半数体特有の外観を呈していた．しかし，尾芽期の末期頃より，尾部の順調な伸長，胸腹部突出の漸減の点で典型的半数体と異な

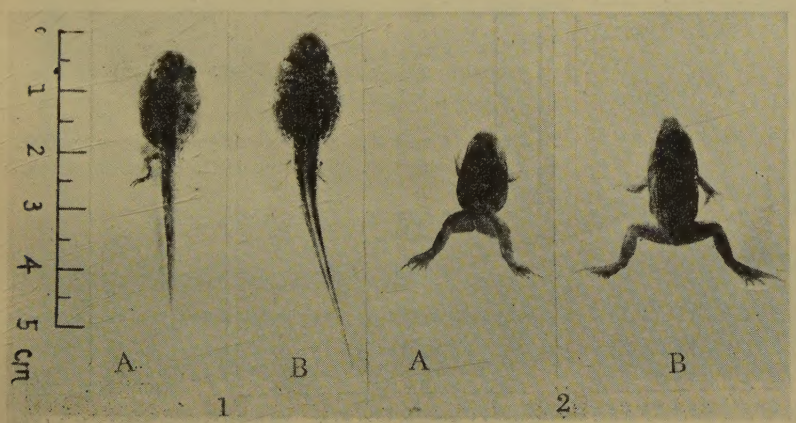


Fig. 1. (A) A haploid tadpole, 84 days old.
(B) A diploid tadpole, 60 days old.
Fig. 2. (A) A haploid frog immediately after metamorphosis.
(B) A diploid frog immediately after metamorphosis.

＊ 採餌し，成長を続ける半数体を，発生初期に水腫を生じて死亡する典型的半数体と区別して，便宜上，生存半数体と呼ぶことにする．

つた発生経過をとり，孵化期前後では両者は明瞭に区別できた．これに対して，生存半数体は，やや小形であるが，むしろ対照区の二倍体と酷似していた．しかしながら，二倍体に比べると，これ等生存半数体の発生速度は，卵黄栓期の頃より徐々に遅れてき，孵化は二倍体より 2～3 日間遅延した．

b) 孵化より変態期迄．生存半数体は孵化後の発育も二倍体より遅れ，また死亡する個体が続出した．尾端法実施後 1 週間内に，被験数の約 60% に当る 56 個体が死亡した．その後の半数体の発育は，個体によつて著しく異り，各個体間の発育速度の差異は，二倍体と発育良好な半数体との間における差異よりも甚だしかつた．

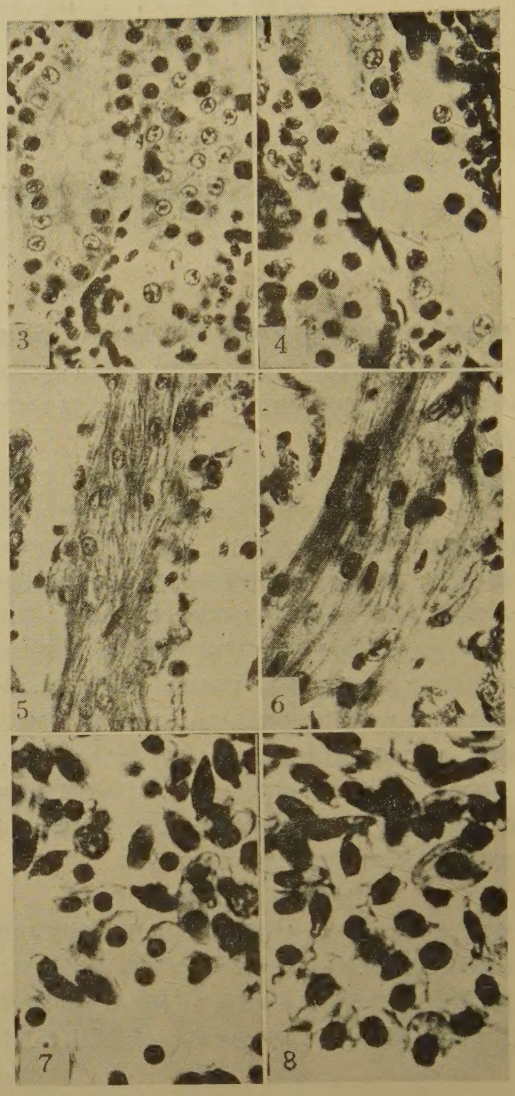
生存半数体のオタマジャクシの外観は，ほぼ同一の発育段階の二倍体に比し，体が小さく，尾がやや短く，吻端及び体全体が丸味を帯びており，眼が小さい等の特徴が見られたが，二倍体に酷似していた（第1図）．しかし，その生活力は著るしく劣弱であり，定量的な測定はできなかつたが，採餌活動も弱く，刺戟に対する反応も鈍かつた．結局，後肢の成長がようやく目立つ頃（日齢 70 日前後）迄生存し得たものは 7 匹であつた．

典型的半数体は，通常，胸腹部に顕著な水腫を生じて死亡するが，生存半数体では殆ど水腫が発生せず，次のような二つの死亡の様相が見られた．即ち，(1) 比較的長期間生存するが，採餌をやめ，腹部が細まり遂に死亡する場合と，(2) 順調に生長を続け，形態的には何等の異常も認められないのに，突然死亡する場合とである．

c) 変態．後肢が発達した7匹のオタマジャクシのうち，4 匹は前肢が出現し，変態期に入つた．二倍体では前肢出現時の日齢は，75～85 日であり，頭胴長は 22 mm 以上に達していたが，半数体では前肢出現は 90～125 日で，頭胴長は 19 mm 前後であつた．また，二倍体では前肢出現時より1週間以内で尾部が完全に吸収され，変態に失敗したものは 14 匹中 2 匹であつたが，半数体は細心の注意を払つたにも拘わらず，変態を完了したものは1匹のみであつた．この半数体は，オタマジャクシ時代から他ものもに比して弱く，成長も遅れ，前肢出現は 110 日齢，尾部の完全吸収には 12 日間を要した．

（2） 半数性幼蛙の特徴

a) 概観．半数性幼蛙は二倍性幼蛙より小さく，特に前後両肢が短小であつた（第2図）．陸上生活における活動は遅鈍を極め，たまたま跳躍すると，その度に仰向に転倒し，独力でもとに戻ることができなかつた．また，下顎の運動による，正規の呼吸運動は，二倍体のように連続的に行われないで，間けつ的に口を開閉するしゃくり状の運動が行われた，腹部及び四肢には，オタマジャクシ時代には全く見られなかつた浮腫が中等度に生じた．この個体は早急に死亡するとも考えられなかつたが，変態完了後5日目にナワシン液で固定した．



Figs. 3～8. Difference in nuclear size between a haploid frog (left column) and a diploid (right column).

Figs. 3 and 4. Sections of the kidneys. ×300.
Figs. 5 and 6. Sections of the heart muscles. ×300.
Figs. 7 and 8. Blood cells. ×760.

開腹の結果，各器官は幾分小さく，軽度の異常（特に肝臓で）が認められたが，根本的な異常は無いように思われた．切片材料の検鏡によつて，各器官及び組織の細胞が小形であること，及び細胞数の増加によつて二倍体の器官の大きさに近ずこうとする補償関係が確められ

た．しかし，赤血球は例外で，その数は非常に少ないように思われた．一般に眼，皮膚，肺臓，腎臓には組織学的な異常は余り見られず，二倍体に類似した分化を遂げていた．骨骼筋は浮腫を生じた為もあつて，筋繊維の走行は疎であり，このことは心筋にも見られた．肝臓の小肝管の分化は，やや不良であり，消化管の分化は比較的良好であつたが，小腸の皺襞の発達は二倍体より若干遅れていた．しかし，一般的について，幼蛙の各器官及び組織は，二倍体に類似した発達分化を示していた．

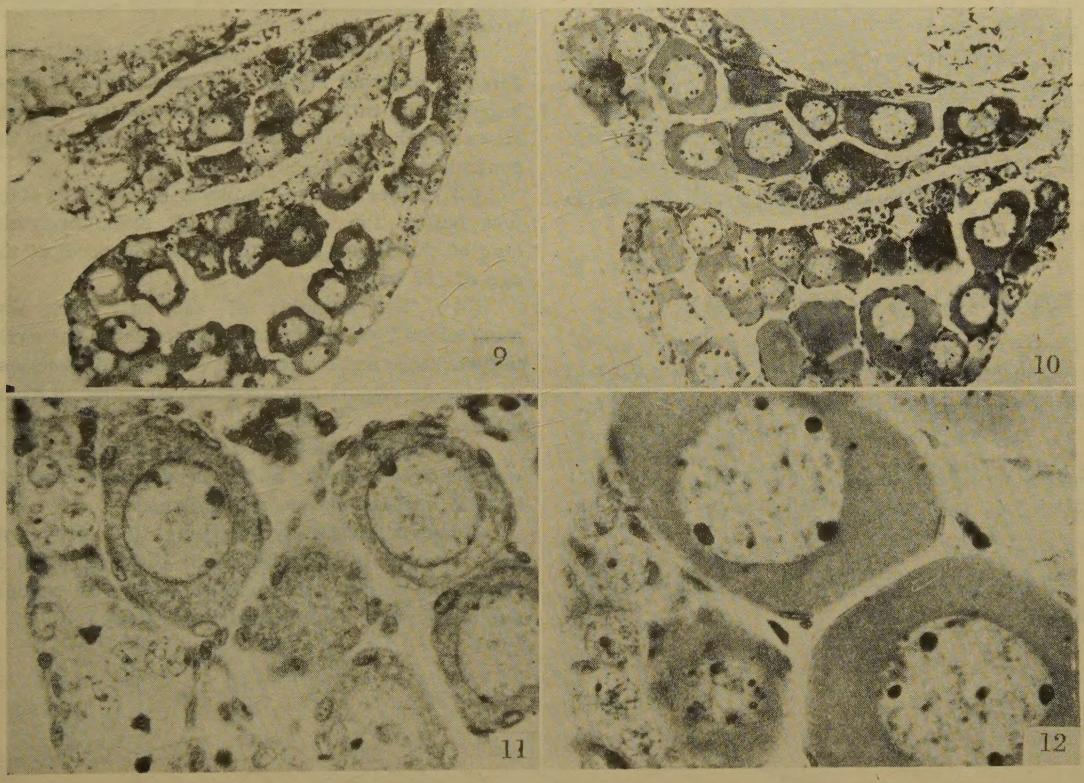
b) 染色体数．　この幼蛙の半数性は，最初尾端法で決定したのであつたが，固定後，各器官及び組織の切片について染色体数を調べた．

網膜，腎臓，皮膚，小腸には比較的明瞭な半数性の中期分裂像が多数存在し，特に皮膚では 13 本の染色体数を算定できた．良効な中期像を認められなかつた器官及び組織では，休止核の大きさと核数を二倍体と比較したが，両者間の差異は低倍率の検鏡でも非常に明確に認められた（第 3～8 図）．特に赤血球が小さく，円形の核を有しているのは印象的であつた（第 7～8 図）．これ等の観察結果から，この幼蛙は純粋な半数体であることが決定できた．

c) 性．　幼蛙の生殖巣は，二倍体よりやや小さく，長さ約 1.2mm（二倍体は約 1.5mm）であつたが，第 9 図に見られるように，典型的な卵巣に分化していた．しかし，卵巣壁がやや肥厚しており，分裂中の多数の卵原細胞が認められた（第 11，12 図比較）．しかしながら，他方，卵巣腔はよく発達しており，第一次卵母細胞の初期のもの及び肥大卵母細胞が極めて多数存在していた（第 9～12 図）．卵原細胞には，染色体数を数え得る程の明確な分裂像が発見できなかつたが，卵巣の半数性は，1) 濾胞細胞の核が小さく，楕円形であり，2) 卵原細胞の休止核が一般に小さく，各種分裂期の染色糸の密度が疎である，3) 肥大卵母細胞が概して小さく，その胚胞内の好酸性斑点の数が少ない等の点で推定できた（第 11 図）．

考　　察

1) 生存半数体の生活力．　生存半数体の生活力は，二倍体より明らかに劣弱であるが，典形的半数体に比較



Figs. 9 and 12. Photomicrographs of sections of ovaries of young frog immediately after metamorphosis.

Fig. 9. The right ovary of a haploid frog. ×95.
Fig. 10. The right ovary of a diploid frog. ×95.
Fig. 11. A haploid frog. ×380.
Fig. 12. A diploid frog. ×380.

すると著しい差がある．生存半数体と典型的半数体との間における，このような生活力の差異は，当然発生に際しての形態及び機能の分化の差異を表わしているものと考えられる．そのような分化上の本質的な原因が，何に由来するかは重要な問題である．Briggs[5]は，生存半数体の器官の核・細胞質比が二倍体と類似していることから，卵自体にオタマジャクシの生活力の差異を生ずる原因があることを指摘した．本実験においても，わづか1匹の雌の卵からのみ生存半数体が発生したという事実は，それらの発生が卵の形態的もしくわ生化学的な構成の特殊性に原因していることを暗示している．この問題については，卵の熟度，環境条件，処理精虫の役割等について更にに詳細な検討が必要であろう．

2）半数体の性．　元来，蛙類の性決定機構は，雌がホモと考えられる．したがつて，卵核発生半数体の性は当然雌が期待される．もちろん，半数性という特殊な生理条件が，比較的不安定な蛙類の性分化に，何等かの影響を与えるかも知れないということは想像に難くない．事実，筆者[2]が先に報告した変態初期に達した半数体には雄性化の傾向が見られた．これに対して，本実験で得られた半数性幼蛙は，若干の異常性が認められたが，よく分化した卵巣を有していた．この卵巣の発育状況，及び変態初期の対照区二倍体が既に明瞭な性分化を遂げている事実を考慮すると，この半数体が，もし生存したとしても，今後雄性化の道をたどるとは到底考えられない．したがつて，この個体は期待に合致した遺伝的な雌であると考えられる．

もちろん，半数体の性決定機構についての詳細は，更に豊富な材料によらないと結論はできない．

要　約

1）トルイヂン・ブルー処理精虫でのダルマガエルの媒精卵から96匹の生存半数性オタマジャクシを得た．

2）生存半数性オタマジャクシの外観は，典型的半数体と異り，尾部が正常に近く，水腫を生じない．また，体が小さく，活動力が鈍く，発生がおくれるという点をのぞけば，むしろ二倍体に酷似している．

3）4匹の半数体が変態期に達し，うち1匹が変態を完了した．

4）変態を完了した1匹の幼蛙は，尾端法及び各器官及び組織の観察によつて，純粋な半数体であることが確かめられた．

5）この半数性幼蛙は雌で，生活力が弱く，各器官及び組織に若干の異常が認められた．しかし，一般的には，二倍体に類似しれ形態的分化を示し，卵巣には肥大卵母細胞が充満していた．

（動物学講座，1957年8月31日受理）

文　献

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Résumé

（1）Ninety-six viable haploid tadpoles of *Rana nigromaculata* HALLOWELL were obtained from eggs which were inseminated with spermatozoa treated with toluidine-blue.

（2）These haploid tadpoles remarkably differed in external appearance from inviable usual haploids and rather resembled the control diploids. However, they were smaller and weaker than the controls and developed more slowly.

（3）Four haploid tadpoles reached the stage just before the completion of metamorphosis. One of them was perfectly metamorposed at the age of 122 days.

（4）The haploidy of the only frog was ascertained both by the tail-tip method in the early tadpole stage and by an observation of the sections of various organs and tissues after killed in fixing fluid.

（5）The haploid frog was a female, which was weak in viability and showed some abnormalities in various organs and tissues.

However, generally speaking, this frog was very similar in morphorogical differentiation to the diploid controls and ovaries filled with numerous auxocytes. (Zoological Laboratory)

ペプシン単分子膜のアルカリによる Expansion について

石井健三・岸本昇三

Expansion of Pepsin Monomolecular Films on the Alkaline Substrate Solutions

Kenzo ISHII and Shozo KISHIMOTO

緒　　論

蛋白質の単分子膜の性質については従来より幾多の実験が行われ，分子の界面における Unfolding による Configuration の変化即ち一つの変性現象として理解されて来た．又単分子膜法による分子量の決定は GUASTALLA[1], BULL[2] 等によつて発展させられ表面変性によつて分裂する様な蛋白も見出された．

又近年種々のポリアミノ酸が合成せられそれを蛋白の模型として用い，より深い知識が得られるようになつた[3]．即ち側鎖に CH_2 を3～4個有し更に適当に -COOH, $-NH_2$ 基を有する両性ポリアミノ酸は特に蛋白質類似の単分子膜を与えている[4][5]．単分子膜に大きな影響を与えるものは Substrate solution の pH 及びイオン強度である．以前は蛋白単分子膜はその等電点附近の pH を持つ溶液上によく拡がり，その両側の pH では極小を有し pH と拡がりの面積との関係は W 型をとるものとされていたが，膜の形成法の改良によつて等電点では最も小なる面積を有することが明らかになつた[6]．

ここでは等電点を3以下の酸性側に有するペプシンについて種々の pH 溶液の場合における分子量の測定と共に膜の性質について研究し，pH が 10 以上のアルカリ側では著しい Expansion を示すことがわかつた．

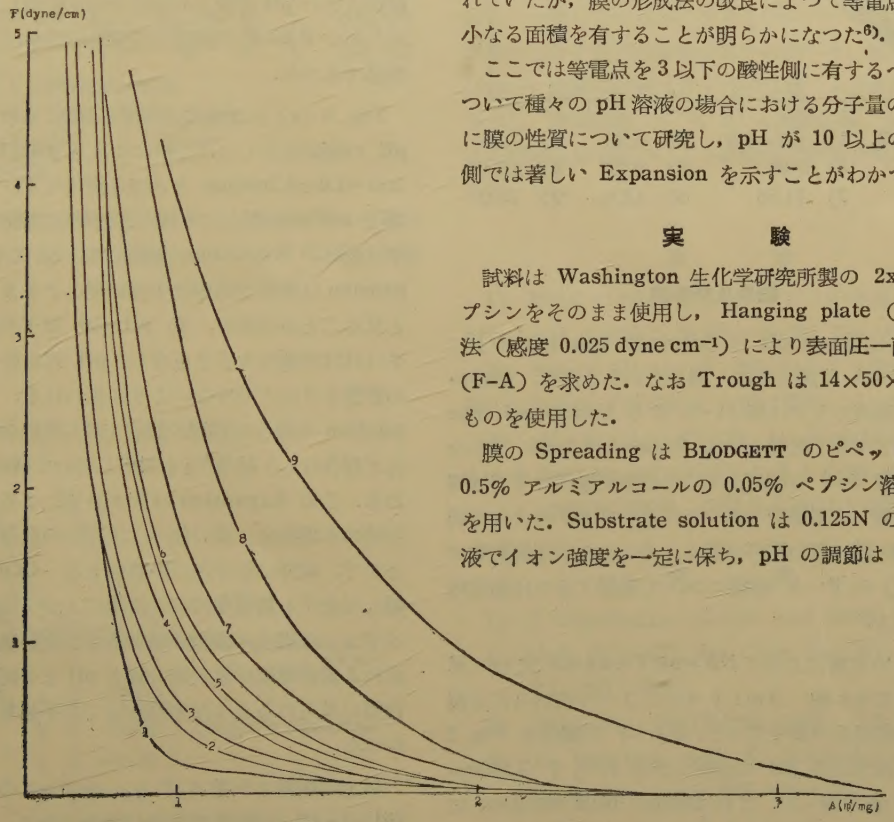

Fig. 1　1) pH=1.9　2) 6.85　3) 7.32　4) 8.95　5) 9.77
6) 10.45　7) 11.58　8) 12.0　9) 13.0

実　　験

試料は Washington 生化学研究所製の 2x-crys. ペプシンをそのまま使用し，Hanging plate（ガラス板）法（感度 0.025 dyne cm^{-1}）により表面圧—面積曲線（F-A）を求めた．なお Trough は $14\times50\times1\ cm^3$ のものを使用した．

膜の Spreading は BLODGETT のピペットを用い，0.5% アルミアルコールの 0.05% ペプシン溶液 0.03cc を用いた．Substrate solution は 0.125N の NaCl 溶液でイオン強度を一定に保ち，pH の調節は HCl 及び

NaOH を使用して特定なイオンを含む Buffer は使用しなかつた．測定は Spreading solution を滴下してから 30 分以上経過し十分平衡になつてから開始した．又測定温度は 6.5±2°C であつた．

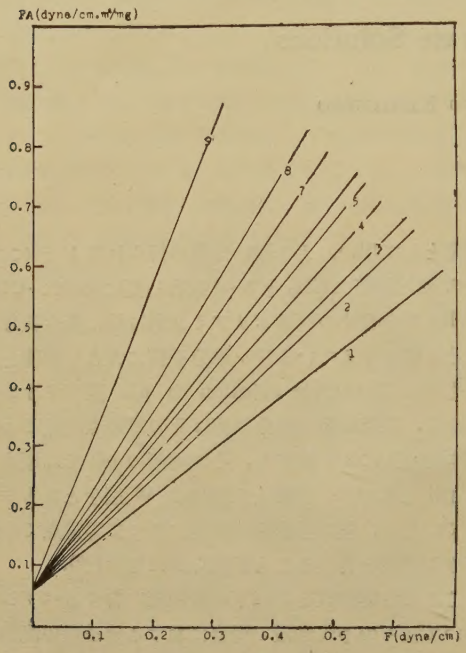

Fig. 2. 1) pH=1.9　2) 6.85　3) 7.32
4) 8.95　5) 9.77　6) 10.45
7) 11.58　8) 12.0　9) 13.0

結果及び考察

Fig. 1 は種々の pH の Substrate solution の上における F—A 曲線である．図からわかる通り，酸性から中性の溶液の上では膜はいわゆる Condensed type であり，アルカリ溶液上では Expanded type であつた．又 pH 4～6 の溶液上ではこの展開条件の下では膜の形成は著しく不安定であつた．そして pH の大なる場合は圧縮の際に膜の溶解が起り，そのため高圧側約数ダイン以上）の F—A 曲線について論議するのは無意味と思われる．

BULL の方法によつて FA=nRT+nAmF 式（n：界面におけるモル数　Am：1 モルによつて占められる面積）から表面分子量を求めた．FA—F の関係を Fig. 2 に示す．結果は各 pH の如何にかかわらず 3,5000～4,0000 の値が得られ，これは従来の Bulk Solution における値，更に BULL の $(NH_4)_2SO_4$ 溶液上，又等電点附近塩酸溶液上の単分子膜の分子量とよく一致してい

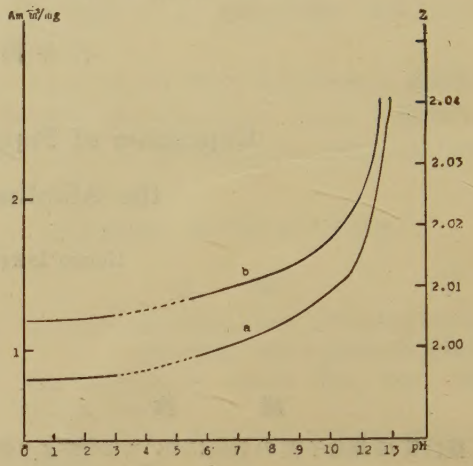

Fig. 3. a) Am—pH relation
b) Z—pH relation

る．[2] 従つてペプシンは等電点から著しく離れ不安定であろうと思われる相当強いアルカリ溶液上でも分裂することなく Spreading していることがわかる．しかも比較的大きい pH 溶液上の場合の方が使用したがガラス板のためか FA—F の関係は再現性が遙かによく F→0 が容易であつた．

Fig. 3 (a) には上式より得られた Am (m²/mg) と pH_- の関係を示した．これによると pH=10 までの間は Am=1.0±0.2m²/mg とさほど変化なく，10 以上になると 2m²/mg 以上にも達して曲線は急激に彎曲し，分子は著しい Expansion を起している．この異常な Expansion は表面変性単分子膜の更にアルカリによる変性と見ることが出来る．又 pH=10 以下ではペプシン分子はほぼ同様な大きさを有し，pH 即ち分子の持つ電荷の影響をさほど受けないように思われる．これは Bulk solution の場合の粘度の測定（即ち酸性中性附近では殆んど変化なし）結果*とも対応していて興味深いと思われる．この Expansion はアルカリによる分子内或は分子間の水素結合，其の他の二次結合の急速な破壊によるもので，同時に分子内に多数有する -COOH 基等が解離して生ずる負電荷同志の反撥によつて促進される．又ペプシンの場合，表面分子のイオン化の程度を示すと言われる表面電位の最大値の値と pH との関係は通常の蛋白質に見られるような中性附近での平坦部が見出されなかつた．

以上の結果より考えると，ペプシンの表面変性は pH=1～10 の範囲では未だ完全でなく，その膜内には

* 未発表

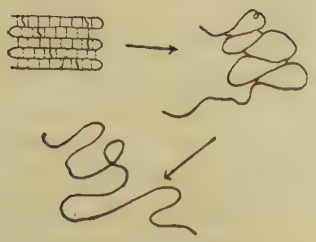

Fig. 4.

その蛋白固有の二次或は三次構造が残存する部分的な表面変性と言える．又この事実は表面電位 △V が pH 小なる場合でも面積の大きい所では比較的小さい値をとり，圧縮によつて急激に大きい値をとることからも推察出来る．

従つて蛋白の Expansion の状態は Fig. 4 のような模型によつて表わされると思う．

さて FLORY, HUGGINS の Polymer solution の理論を Polymer の単分子膜に適用した SINGER の状態方程式[7]

$$F=-\frac{kT}{A_0}\left[\ln\left(1-\frac{A_0}{A}\right)-\frac{(x-1)}{x}\frac{Z}{2}\ln\left(1-\frac{2A_0}{ZA}\right)\right]$$

A：アミノ酸残基の面積
A_0：アミノ酸残基の実際の面積
x：一分子中のアミノ酸の数
Z：配位数

は DAVIES によつて両性ポリアミノ酸或は種々の蛋白質に適用され有益な結果が得られている．[5)8)] 種々の場合 Z は 2～4 の値をとり，DAVIES は Z—2 を Degree of unfolding と呼んでいる．

$A_0/A \ll 1$ の場合に

$$FA=\frac{kT}{x}\left[1+\frac{A_0}{A}\left(\frac{2-2x+Zx}{2Z}\right)\right]$$

により求めた Z の値と pH との関係は Fig. 3 (b) に表わされた通りであるが，Am—pH 曲線と全く一致したものが得られ，どちらも分子の unfolding に密接に関係していることが明瞭である．

一般に表面圧に表われる要素は Kinetic energy の他に Polymer の分子内及び分子間の反撥の Energy である．DAVIES によれば分子内の Energy は free の -COOH, $-NH_2$ 等の持つ電荷による力と側鎖の持つ van der Waals 力によつて Z で規定される．従つて実際の理論的な状態式は Singer の方程式に更に分子間力を含む項を附加せねばならない．このような議論を今のペプシンの単分子膜に当てはめ，高面積即ち分子間力が無視されると思われる場合について求めた Z を用いて Singer の式から計算した F—A を書くと Fig. 5 (a, b, c) である．即ち等電点にほぼ近く，電荷も小さい場合，例えば pH 1.9 の場合には理論曲線 (Z=2.05) によく一致しているがアルカリ側にいく程，大きな面積の所から分子間力の影響が現われ始める．即ち a—b 及び a—c 曲線の間隔は分子内の反撥による表面圧の増加を示し，理論曲線 (a, b, c) と実験曲線 (a′, b′, c′) との間隔は分子間力による表面圧の増加を表わすものである．

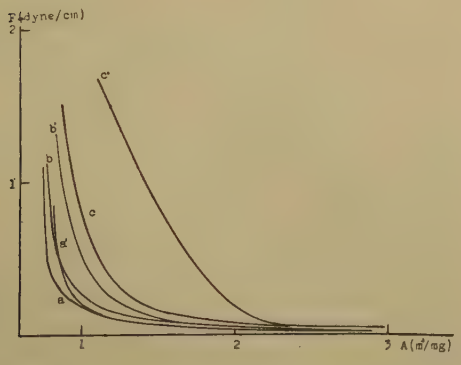

Fig. 5. a Z =2.05, b Z =2.10, c Z =2.30 } Singer's equation
a′ pH=1.9, b′ pH=7.32, c′ pH=12.0 } Experimental

結　　論

1) BULL の方法で求めたペプシンの表面分子量は pH=1～13 の範囲で 35,000～40,000 であつた．

2) Am は pH=1～10 の間ではあまり差がなくこの間では分子の Unfolding の度は電荷によらない．

3) pH=10 以上の膜は著しい Expansion を示し Am も SINGER の式から求めた Z も急激に大きくなる．

（物理化学講座，1957 年 8 月 31 日受理）

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Summary

The effect of pH upon F-A relations with respect to unimolecular films of pepsin is investigated at the air/water interface in the constant ionic strength.

The calculations of surface molecular weight of pepsin and of the area (Am) of films in the gaseous state by Bull's method, give results that the surface molecular weight of pepsin has no change at different pH, while Am is increases in a marked degree at pH 10 and upwards.

Finally, applying the Singer equation to these films, we have calculated the flexibilities of this molecule and discussed qualitatively the intra-inter molecular forces of these films.

(Laboratory of Physical Chemistry)

A DOUBLE THIN LENS BETA-RAY SPECTROMETER WITH RING FOCUS III.

OPTIMUM SOURCE SIZE AND CONE BAFFLE

K. Takazawa, M. Higuchi, K. Ohira and H. Matsui

In a double thin lens beta-ray spectrometer with a Hubert slit system, the effects of the finite source size are studied theoretically and experimentally, and the optimum source sizes for a given emission angle are determined. A cone baffle has been used and it is found that the relation between luminosity and resolution for the optimum source size is considerably improved, in particular, for higher resolutions. This improvement makes it possible to use practically the instrument with considerably high resolution, 0.5%.

1. Introduction

Though many thin lens spectrometers have been constructed and widely employed,[1)-10)] not very much have been published on the theory of this type of spectrometers, because of the difficulty of the analytical treatment. Therefore, it will be necessary to study this type of spectrometer in detail in order to improve its performance.

A simplified theory of the thin lens spectrometer has been given by Deutsch, Elliott, and Evans,[1)] but it is confined to the case of axial focusing. Keller, Koenigsberg, and Paskin[11)] have discussed the existence of the ring focus and the effects of source size by the numerical integration of a differential equation, but neither the optimum emission angle nor the optimum source size is discussed. Perkins and Solbrig[12)] have proposed the graphical method and determined the most effective baffle arrangement for a point source, the so-called Hubert slit system.[13)]

In our previous paper I,[14)] the design and the performance of a double thin lens beta-ray spectrometer have been described. In our paper II,[15)] extending the graphical method to the case of the finite source, the effect of the source size has been discussed and hence the more effective baffle arrangement for the finite source, the cone baffle, has been suggested. The effort has been made to obtain the optimum emission angle, in paper I, and it was shown that the optimum angle would be far larger than 10°. But because of the tedious calculations the optimum source sizes were not obtained.

Since the thin lens spectrometer is not different essentially from the homogeneous one, the method used and even the qualitative results obtained in the latter will be applied to the former case. In this paper, it is shown that the simple folding method used by DuMond[16)] when he discussed the optimum conditions in the homogeneous field spectrometer is successfully applied to our spectrometer. Using this method the optimum source sizes are determined and compared to the experimental results. The cone baffle suggested in paper II has been used and the expected improvement of the performance is obtained.

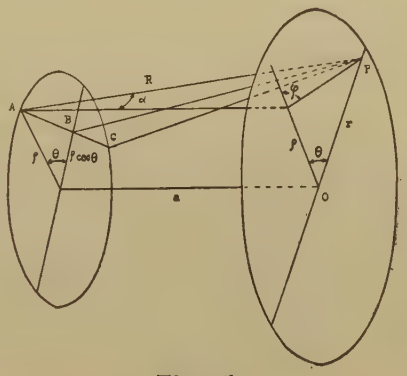

Fig. 1.

2. Basic Consideration

Combining the numerical method described by Keller et al.[11)] and the graphical one proposed by J. F. Perkins and A. W. Solbrig, Jr.,[12)] the line profiles for a point source are determined. The baffle system adopted is a so-called "Hubert slit system" and the high energy side of the line profile is sharply cut off. Extending the graphical method to the case of a finite source size and using the appropriate approximations adopted by Keller et al., any ray originating from any point on the source is obtained. The detail of the method was given in paper II. Using the notation of

Fig. 1, the transmission T is defined by

$$T=\frac{2\pi\iiint\frac{r\,dr\,d\theta}{R^2}\cdot\frac{a}{R}\rho\,d\rho}{4\pi\ (\pi\rho_1^2)}$$

$$=\frac{1}{2\pi\rho_1^2}\int_0^{\rho_1}\rho\,d\rho\int_0^{2\pi}d\theta\left\{A_1\cos\alpha_1-A_2\cos\alpha_2\right\},\quad(1)$$

$$A_1=\frac{a^2+\rho^2-r_1\rho\cos\theta}{a^2+\rho^2-\rho^2\cos^2\theta},\quad A_2=\frac{a^2+\rho^2-r_2\rho\cos\theta}{a^2+\rho^2-\rho^2\cos^2\theta},$$

where r_1 and r_2, the extreme values of r, are the functions of ρ, θ, and k (momentum in gauss cm/amp.). It must be noted that the integrand of eq. (1) depends on ρ, θ, and k. As regards this dependency it will be discussed in section 4. The numerical calculations of this integral being very tedious, the line profiles were obtained only for a certain gathering power in paper II. In order to obtain the optimum source size, it is necessary to know a large number of line profiles for various gathering powers and source sizes and therefore the simpler approximations must be introduced to avoid the tedious calculations. For this purpose it seems to be worthwhile to recall what kind of approximation was used for the angular momentum ray in paper II.

In Fig. 1, we consider a ray starting from a point $(\rho, 0, 0,)$ and passing through a point $(r_0, \theta, a,)$ in cylindrical coordinate. In the region where the field is sufficiently weak, the ray is approximately straight line in space and the $(r-z)$ curve of the ray appears as a hyperbola. In the region of the strong field, the effect of angular momentum is small and the trajectory of angular momentum may be considered to be nearly same to the one of zero-angular momentum. Thus the former is replaced by the latter originating from the point where the tangent of the hyperbola at some point sufficiently apart from the source intersects with the source plane. The equation of the above hyperbola is

$$r^2=\left(\frac{z}{a}\right)^2\left\{r_0^2-2r_0\rho\cos\theta+\rho^2\right\}$$

$$+\frac{2z\rho}{a}(r_0\cos\theta-\rho)+\rho^2.\quad(2)$$

The gradient at $z=a$ is

$$\left(\frac{dr}{dz}\right)_{z=a}=\frac{1}{a}(r_0-\rho\cos\theta).\quad(3)$$

If the tangent of the hyperbola at $z=a$ intersects with the source plane at a point whose radial position is ρ_0,

$$\frac{r_0-\rho_0}{a}=\frac{1}{a}(r_0-\rho\cos\theta),$$

or

$$\rho_0=\rho\cos\theta.\quad(4)$$

Therefore, the trajectory of an angular momentum ray starting from $(\rho, 0, 0,)$ and passing through $(r, \theta, a,)$ is replaced by the one of a zero angular momentum ray starting from a point $(\rho\cos\theta, \theta, 0,)$ and passing through the same point. Furthermore, all rays with a given momentum starting from the points on the line AC and passing through a point $(r, \theta, a,)$ have the same trajectory and focus to the same radial position at the ring slit. This is just the same approximation used by DuMond when he discussed the optimum conditions for a homogeneous field beta-ray spectrometer.

DuMond[16] assumed three independent instrumental sources of line spread, (1) the spherical aberration coming from the use of a finite angular aperture, (2) the finite width of the ring slit, (3) the finite source size. Each of these sources of line spread yields its own characteristic spectral line profile. As shown in Fig 2, the line profiles I, II and III introduced by the above sources of line spread are the inverse half profile, the rectangular profile and the elliptical profile, respectively. A_3 is the half-width of the profile III at the base. Folding the profiles I and II, the line

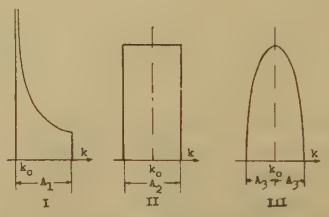

Fig. 2. Three characteristic line profiles I, II, and III. (DuMond, reference 16)

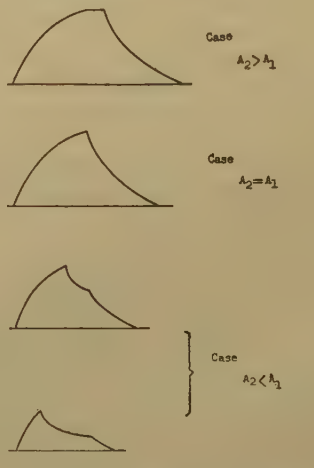

Fig. 3. Form of the line profile for a solenoid spectrometer with point source. A_1 aberration width, A_2 width of the exit slit. (DuMond, reference 16.)

profile for a point source is obtained. The form of the line profile for the different angular apertures and slit widths is shown in Fig. 3. When the slit width is larger than the corresponding optimum angular aperture, the profile has a flat top. When the slit width is smaller than the corresponding optimum angular aperture, the slope of the profile becomes discontinuous at the higher energy side. Hence the optimum angular aperture is obtained for a given slit width. Folding the profile for a point source and the profile III, the profile for a finite source is obtained.

In our case, the profiles for a point source are obtained by the graphical method and shown in Fig. 4. The profiles for various silt widths have, due to the Hubert slit system, neither flat tops nor discontinuities of slope at the higher energy side. This means that the Hubert slit system always selects the optimum angular aperture automatically as the slit width is varied. As DuMond pointed out in his paper, the procedure of choosing first the optimum condition for a point source and folding then the profile III, is strictly open to criticism. It seems likely, however, that no significant improvement would be obtained by modifying it.

In our double thin lens spectrometer, A_3 is a complicated function of the source radius ρ, the emission angle α, and the electron momentum k. This is shown in Table I.

Owing to the above dependency of A_3 on α and k, the profile III is not an ellipse, and the procedure of folding the three profiles becomes invalid. The validity of the folding procedure depends on the mutual independence of three separate sources of line spread, and in the homogeneous field spectrometer this independency is assumed in the first approximation by DuMond.

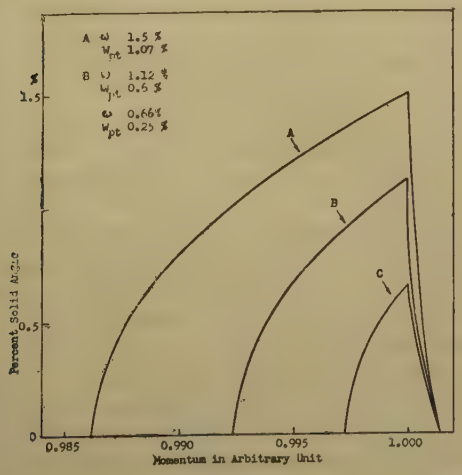

Fig. 4

Table I

ρ cm	k gauss cm/amp.	α	A_3 gauss cm/amp.
0.225	1420	16°	10.0
0.225	1420	12°41′	12.86
0.225	1420	10°46′	15
0.225	1420	8°	20
0.225	1400	16°	11.2
0.225	1400	12°41′	14.2
0.225	1400	10°46′	16.3

Recently, however, D. B. Beard[17] has pointed out the invalidity of this folding process, showing that the profile II is a function of ρ and not a step function for a finite ρ. As regards this criticism the readers will be refered to section 4.

For simplicity, however, A_3 is assumed to be independent of α and k and linearly dependent on ρ. This seems to be a good approximation for small source size and slit width, but for ρ=0.225cm it becomes to be very crude, as seen from Table I. A_3/ρ is computed for k=1420 gauss cm/amp. and α=12°41′.

It is interesting to note that the source spread is decreased as the emission angle increases. The gathering power for a given $\Delta\alpha$ increases of course as α increases. From the both points of view, the optimum emission angle must be larger than at least 16°.

3. Optimum Source Size

The resultant line profiles for various ρ and slit widths are obtained by the simple folding procedures. Since W_{pt}, the half-maximum width for a point source, is simply related to the slit width on account of Hubert slit system, W_{pt} will be used hereafter instead of the slit width.

The resultant half-maximum widths W obtained for various W_{pt} and A_3 are shown in Table II.

Table II

gathering power %	W_{pt} gauss cm/amp	%	A^3 gauss cm/amp	source dia. mm	W gauss cm/amp	%
0.66	4	0.27	5.7	2.0	9.266	0.65
0.66	4	0.27	4	1.4	6.7	0.47
0.66	4	0.27	3	1.05	5.1	0.36
0.66	4	0.27	2	0.7	4.4	0.31
1.5	15.9	1.12	15	5.3	24.85	1.76
1.5	15.9	1.12	13	4.5	22.2	1.57
1.5	15.9	1.12	9	2.8	18.4	1.30
1.5	15.9	1.12	5.7	2.0	17.0	1.20

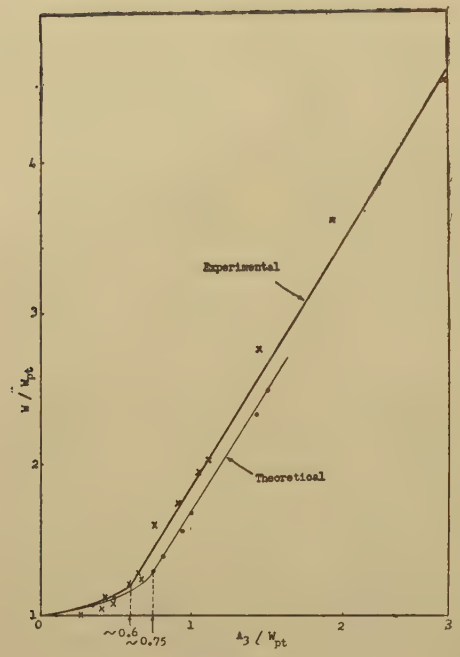

Fig. 5

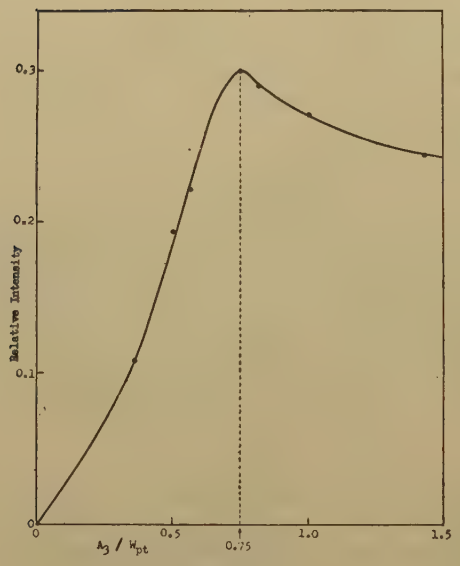

Fig. 6

If we plot W (represented in terms of W_{pt}) as a function of A_3/W_{pt}, we obtain a fairly smooth curve as shown in Fig. 5. From this figure, we can express W_{pt} and A_3 in terms of W for various ratios A_3/W_{pt}. Now the luminosity L is defined by

$L=\omega\sigma$. ω: gathering power
σ: source area

Since ω is proportional to $W_{pt}^{1/2}$ and σ to A_3^2, L will be proportional to the product $W_{pt}^{1/2}A_3^2$. Thus it is possible to compute L for constant W as a function of A_3/W_{pt}. This is shown in Table III and Fig. 6. It is easily seen that the optimum value of A_3/W_{pt} is 0.75, and the corresponding W is $1.29W_{pt}$. For the optimum source width $A_3=0.75W_{pt}$ the luminosty L is proportional to $W^{5/2}$ as in the solenoid type.

Table III

A_3/W_{pt}	W	W_{pt}	A_3	$W_{pt}^{1/2}A_3^2$
1.425	$2.332W_{pt}$	$0.611W$	$0.429W$	$0.245W$
1.00	1.686	0.593	0.593	0.271
0.8175	1.396	0.586	0.716	0.291
0.75	1.286	0.583	0.778	0.300
0.566	1.16	0.488	0.862	0.221
0.5	1.107	0.452	0.903	0.194
0.3585	1.069	0.335	0.936	0.109

Using K-conversion electrons from Cs^{137}, the resultant half-widths W were obtained for five different source diameters and three different slit widths. The source was prepared on mica backing of ~1 mg/cm² thickness by evaporation of the droplets using insulin as the spreading medium. The source thickness was less than 0.1 mg/cm². The great care has been taken to locate the center of the source on the axis of the instrument as precisely as possible.

The results are shown in Figs. 5 and 7. Though small discrepancy is seen between theoretical and experimental curves, in Fig. 5, they are in rough agreement. This discrepancy comes from the crude approximation used for the line profile III, which neglects the dependency of A_3 on α and k. If we take somewhat larger value for A_3/ρ, better agreement will be obtained. The slope of the theoretical curve changes considerably at the point $A_3/W_{pt}=0.75$, which corresponds to the optimum values of the ratio A_3/W_{pt} as described above. From the similar point for the experimental curve it is seen that the practical optimum ratio A_3/W_{pt} is about 0.6. In this case the resultant half-maximum width W is about $1.25W_{pt}$.

In Fig. 7, the relative peak intensity *vs* half-maximum width W is plotted for a given source diameter and three different slit widths. Similar curves for various source diameters are also shown. The envelope of these curves represents the relation between the relative intensity and W for the optimum source size. From the above relations and curve we can easily determine the optimum source diameter and slit width for a given W.

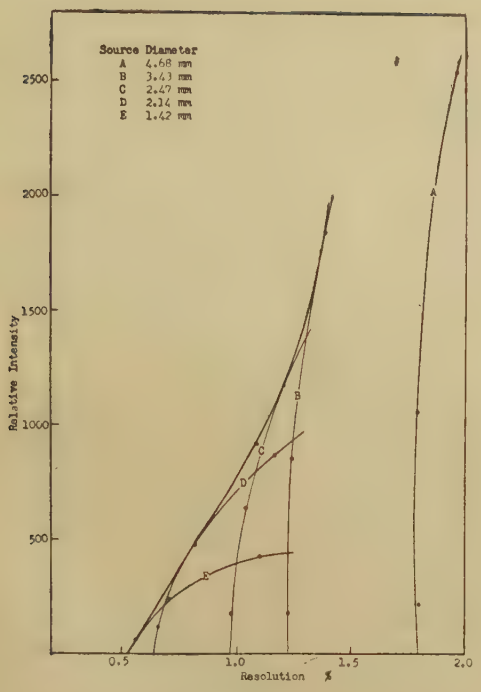

Fig. 7

4. Cone Baffle

In paper II, it has been shown that the rays of desired momentum originating from the extended source are not fully utilized, while the undesired rays of somewhat different momentum are admitted more than the desired. The utilizable portions of the defining aperture are illustrated in Fig. 9 of paper II for two extreme cases. The lower portion of the aperture for the desired rays and the upper portion for the undesired are not utilized. Hence the use of a cone baffle with vertex at the center of the source and axis coincident with the axis of the instrument has been suggested.

D. B. Beard,[17] in a homogeneous field spectrometer, has pointed out the effect on the line profile shape and intensity of the angle φ, the angle made by the projection of the electron velocity vector on a plane perpendicular to the instrument axis at the source with the radius of the source point. He estimated correctly the effect of the source size and showed that the spread due to the source size is $\rho\cos(\varphi+\Psi)$, where Ψ is the angle of rotation at the ring slit. The transmission or $\Delta\alpha$ permitted by the slit system is, then, a function of k, α, ρ, and φ, and not all values of φ are permitted. Therefore, besides three diaphragms controlling Δk, $\Delta\alpha$, and ρ, the fourth diaphragm restricting φ must be taken into consideration. Since the spread due to the source size is $\rho\cos(\varphi+\Psi)$, ρ may be appreciably larger when the spread due to φ is restricted by φ-selecting baffle. From this point of view, Beard has shown that the improvement might be a factor of 10 by the use of the appropriate φ-selecting baffle and source size.

In DuMond's folding method, it seems to be considered that the transmission or $\Delta\alpha$ permitted by the slit system depends on φ. The elliptical line profile III comes from the approximation that the angular momentum ray starting from the radial position ρ on the source is replaced by the zero-angular momentum one starting from the radial position $\rho\cos\theta$ as discussed in section 2.

In this approximation, the trajectory for given α, k, and ρ varies with θ. This is nothing but the θ-dependency (or φ-dependency) of the transmission. However, the φ-dependency is not estimated correctly in this approximation because the spread due to the source size is not $\rho\cos\theta$ but $\rho\cos(\varphi+\Psi)$ as shown by Beard. In DuMond's paper, the φ-dependency has been expressed implicitly in the elliptical line profile III, while, in our paper II, this dependency has been considered explicitly though rather incorrect, and hence the cone baffle, a kind of φ-selecting diaphragm, has been suggested.

The role of the cone baffle may be understood from another point of view: Since the rays starting from the source and passing through a certain point of the defining aperture will be partially stopped by the cone baffle, the profile III will be asymmetric and resemble to the asymmetric line profile for a point source with the Hubert slit system. This will decrease the half-maximum width of the resultant line profile without the serious reduction of the intensity.

As shown in Fig. 8, an aluminum cone of angle 17°31′, machined stepwise to reduce scattering, is placed so as to graze the innermost rays originating from the center of the source. Using the same sources described in section 3, the effect of the cone baffle was studied experimentally. The line profiles obtained with the cone baffle for 4.68 mm dia. source are shown and compared to the one without the cone baffle in Fig. 9. As expected, the half-widths are greatly decreased without any reduction of intensity. But since 4.68 mm source is greater than the optimum source size for the slit widths used and the effective source width may be decreased by the use of the cone baffle, the relations between intensity and resolution for the optimum source sizes with and without the cone baffle must be compared. The intensity *vs* resolution curve for the optimum source sizes with the cone baffle is obtained in the similar manner described in section 3, and

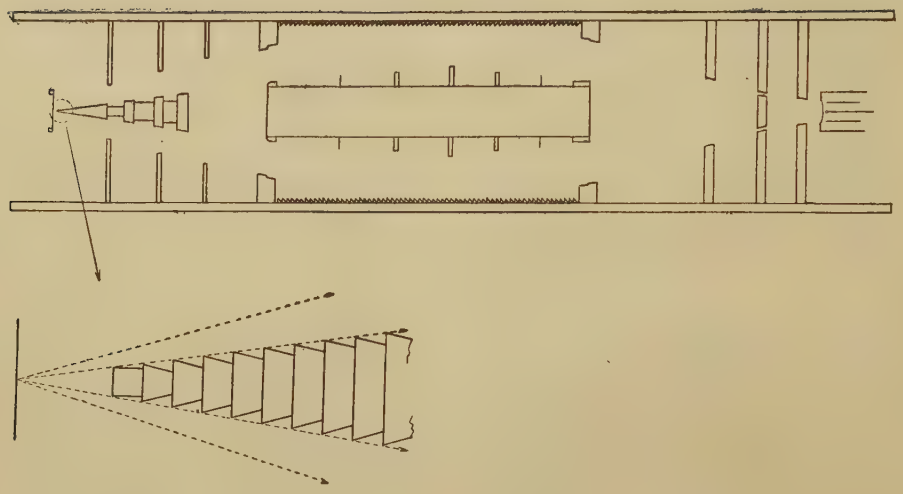

Fig. 8

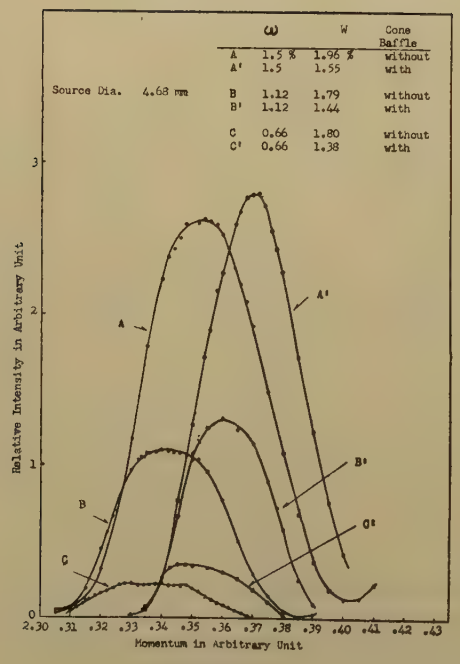

Fig. 9

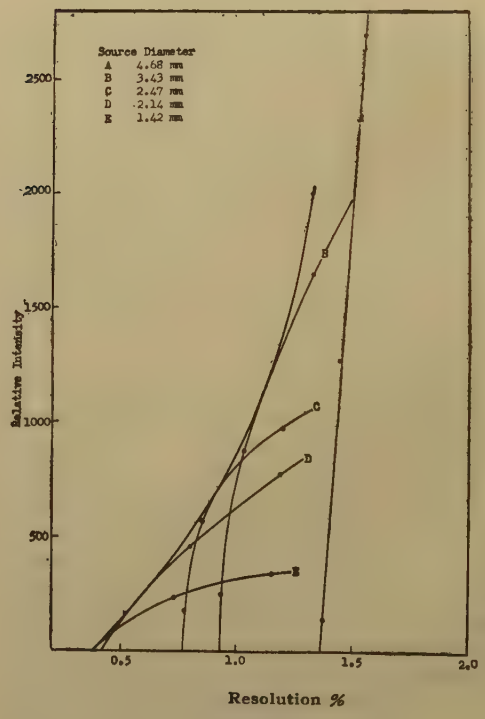

Fig. 10

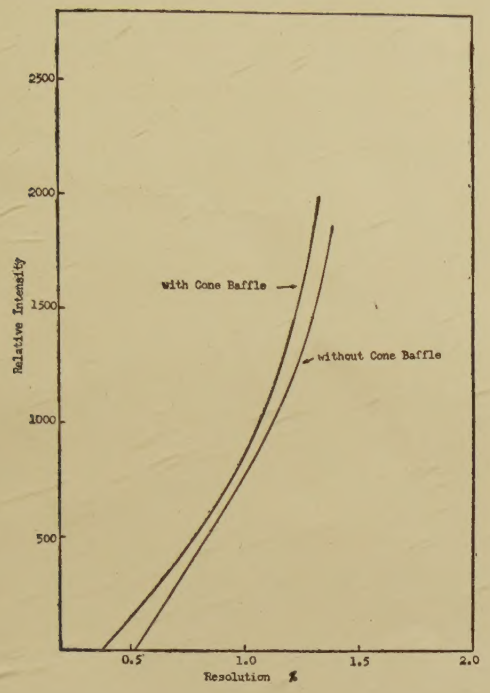

Fig. 11

shown in Fig. 10. In Fig. 11 the curves with and without the cone baffle are compared. From this figure it is clearly seen that the considerable improvement is attained, in particular, for higher resolutions. This improvement makes it possible to use practically the instrument with considerably high resolution, 0.5%.

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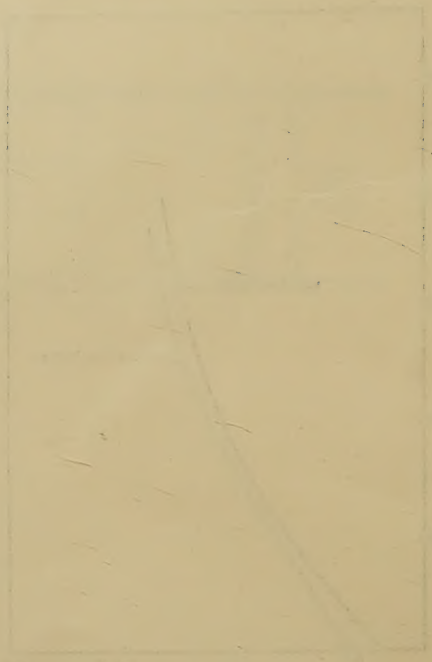
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